



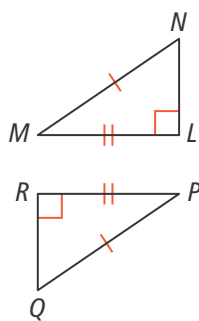
UNDERSTAND

9. **Construct Arguments** Follow the steps to prove the HL Theorem.

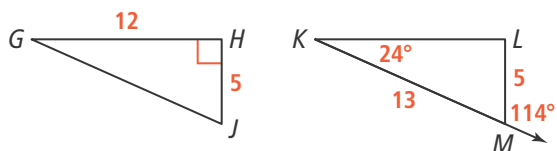
Given: Right triangles $\triangle MNL$
and $\triangle PQR$, $\overline{MN} \cong \overline{PQ}$,
 $\overline{ML} \cong \overline{PR}$

Prove: $\triangle MNL \cong \triangle PQR$

- Show that there is a rigid motion that maps L to R and M to P so that N' and Q are on opposite sides of $\overleftrightarrow{PR}$.
- Then show that $\triangle PQN'$ is isosceles.
- Show that $\triangle M'N'L' \cong \triangle PQR$,
so $\triangle MNL \cong \triangle PQR$.

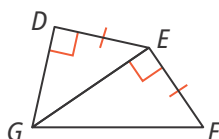


10. **Mathematical Connections** Consider the figures.



Describe the steps you would have to take before you could use the HL Theorem to prove $\triangle GHJ \cong \triangle KLM$.

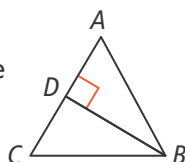
11. **Error Analysis** Mohamed wrote the paragraph proof to show that $\triangle DEG \cong \triangle EFG$. What mistake did he make?



$\triangle DEG$ and $\triangle EFG$ are right triangles.
The figure shows $\overline{DE} \cong \overline{EF}$, AND
 $\overline{EG} \cong \overline{EG}$ by the Reflexive Property.
Therefore, by the HL theorem,
 $\triangle DEG \cong \triangle EFG$.



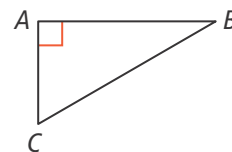
12. **Higher Order Thinking** Suppose $\triangle ABC$ is an equilateral triangle. Use the HL Theorem to explain why any segment perpendicular to a side from the opposite vertex produces two congruent triangles. Would the same be true if $\triangle ABC$ were an isosceles triangle that was not equilateral? Explain.



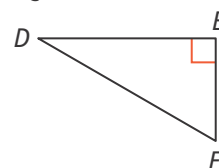
PRACTICE

For Exercises 13–16, you are given a theorem and a congruence statement. What additional information is needed to prove that the triangles are congruent? SEE EXAMPLE 1

13. By using ASA, given
 $\overline{AC} \cong \overline{EF}$



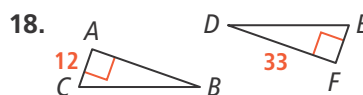
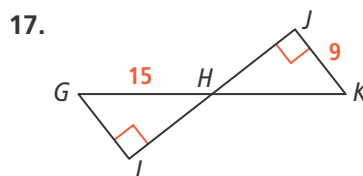
14. By using AAS, given
 $\angle B \cong \angle D$



15. By using SAS, given
 $\overline{AB} \cong \overline{DE}$

16. By using HL, given
 $\overline{CB} \cong \overline{DF}$

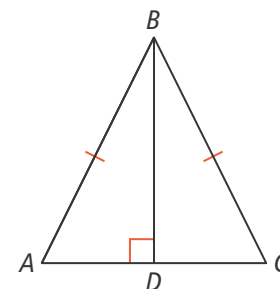
For Exercises 17–18, what information would be sufficient to show that the triangles are congruent by the HL Theorem? SEE EXAMPLE 2



For Exercises 19–20, write a proof using the HL Theorem to show that the triangles are congruent. SEE EXAMPLE 3

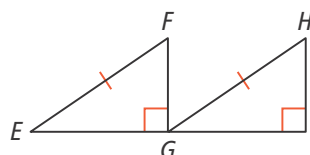
19. Given: $\overline{AB} \cong \overline{CB}$,
 $\overline{AC} \perp \overline{DB}$

Prove: $\triangle ABD \cong \triangle CBD$



20. Given: $\overline{EF} \cong \overline{GH}$, G is the midpoint of $\overline{EJ}$

Prove: $\triangle EFG \cong \triangle GHJ$

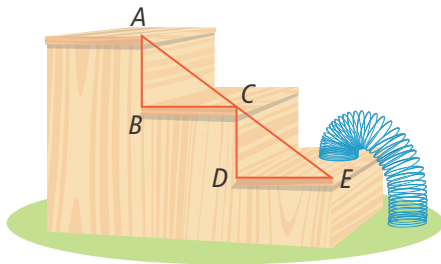


APPLY

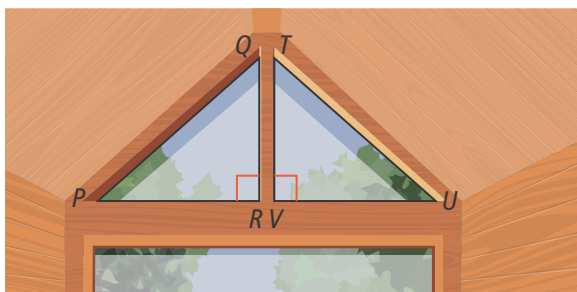
21. **Make Sense and Persevere** Part of a truss bridge consists of crossbeam $\overline{KM}$ and a perpendicular beam $\overline{LN}$. What beams could an engineer measure in order to show $\triangle KLN \cong \triangle MLN$ using the HL Theorem?



22. **Construct Arguments** Raul wants to verify that the steps built by a carpenter are uniform by checking that $\triangle ABC \cong \triangle CDE$. The carpenter assures him they are because $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DE}$.
- Explain why Raul cannot use the HL Theorem to prove $\triangle ABC \cong \triangle CDE$.
 - Is there another theorem that Raul can apply to prove $\triangle ABC \cong \triangle CDE$? If so, state the theorem. If not, explain why not.

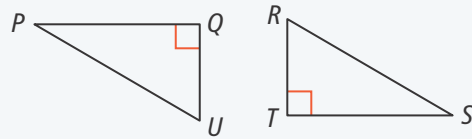


23. **Communicate Precisely** What are the fewest measurements that a homeowner could make to be certain that the front windows shown below are congruent?



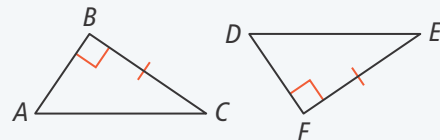
ASSESSMENT PRACTICE

24. Match each set of congruence statements with the theorem that can be used to prove that the two triangles are congruent.



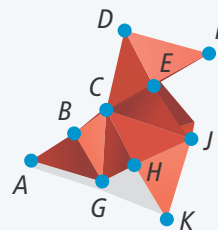
- | | |
|---|--------|
| I. $\overline{PQ} \cong \overline{ST}$ and $\overline{QU} \cong \overline{TR}$ | A. ASA |
| II. $\overline{PU} \cong \overline{SR}$ and $\overline{QU} \cong \overline{TR}$ | B. AAS |
| III. $\overline{QU} \cong \overline{TR}$ and $\angle U \cong \angle R$ | C. SAS |
| IV. $\overline{QU} \cong \overline{TR}$ and $\angle P \cong \angle S$ | D. HL |

25. **SAT/ACT** Which statement proves the triangles are congruent using the HL Theorem?



- | | |
|---------------------------------------|---------------------------------------|
| Ⓐ $\angle A \cong \angle D$ | Ⓒ $\angle B \cong \angle F$ |
| Ⓑ $\overline{AB} \cong \overline{DF}$ | Ⓓ $\overline{AC} \cong \overline{DE}$ |

26. **Performance Task** Holly makes the origami figure shown. Assume that every angle that appears to be a right angle is a right angle.



Part A What can Holly measure so that she can use the HL Theorem to prove that $\triangle ABG \cong \triangle CBG$?

Part B Holly measures to find that $HK = DE$ and $HJ = EC$. Is it possible for her to apply the HL Theorem to prove that $\triangle JHK \cong \triangle CED$? Explain.

Part C Choose two other triangles on the figure. Describe what Holly could measure to prove the triangles are congruent by using the HL Theorem.