STUFF YOU MUST KNOW COLD . . .

Alternate Definition of the Derivative:

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

Basic Derivatives

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$$

Where u is a function of x, and a is a constant.

Differentiation Rules

Chain Rule:

$$\frac{d}{dx}[f(u)] = f'(u)\frac{du}{dx} \quad OR \quad \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Product Rule:

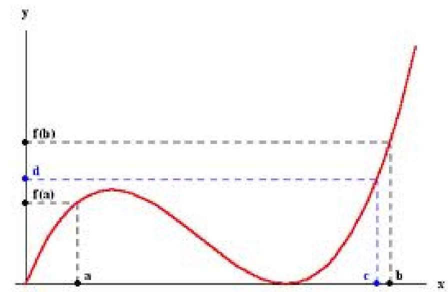
$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx} \quad OR \; u\,v' + v\,u'$$

Quotient Rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \quad OR \quad \frac{v\,u' - u\,v'}{v^2}$$

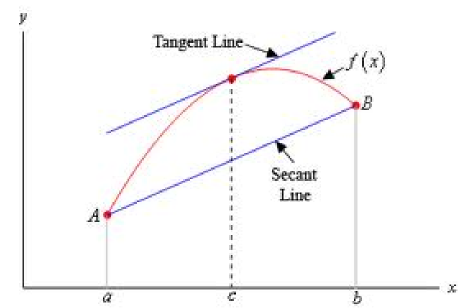
Intermediate Value Theorem

If the function $f(x)$ is continuous on $[a, b]$, and y is a number between $f(a)$ and $f(b)$, then there exists at least one number $x = c$ in the open interval (a, b) such that $f(c) = y$.



Mean Value Theorem

If the function $f(x)$ is continuous on $[a, b]$, **AND** *__the first derivative exists on the interval__* (a, b) then there is at least one number $x = c$ in (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

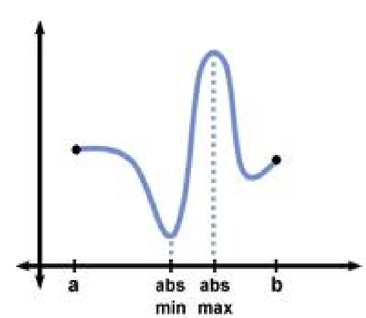


Rolle's Theorem

If the function $f(x)$ is continuous on $[a, b]$, **AND** *__the first derivative exists on the interval__* (a, b) **AND** $f(a) = f(b)$, then there is at least one number $x = c$ in (a, b) such that $f'(c) = 0$.

Extreme Value Theorem

If the function $f(x)$ is continuous on $[a, b]$, then the function is guaranteed to have an absolute maximum and an absolute minimum on the interval.



Derivative of an Inverse Function:

If f has an inverse function g then:

$$g'(x) = \frac{1}{f'(g(x))}$$

derivatives are reciprocal slopes

Implicit Differentiation

Remember that in implicit differentiation you will have a $\frac{dy}{dx}$ for each y in the original function or equation. Isolate the $\frac{dy}{dx}$. If you are taking the second derivative $\frac{d^2y}{dx^2}$, you will often substitute the expression you found for the first derivative somewhere in the process.

Average Rate of Change ARoC:

$$m_{sec} = \frac{f(b) - f(a)}{b - a}$$

Instantaneous Rate of Change IRoC:

$$m_{tan} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Curve Sketching And Analysis

$y = f(x)$ must be continuous at each:

Critical point: $\frac{dy}{dx} = 0$ or undefined

LOOK OUT FOR ENDPOINTS

Local minimum:

$\frac{dy}{dx}$ goes $(-, 0, +)$ or $(-, und, +)$ OR $\frac{d^2y}{dx^2} > 0$

Local maximum:

$\frac{dy}{dx}$ goes $(+, 0, -)$ or $(+, und, -)$ OR $\frac{d^2y}{dx^2} < 0$

Point of inflection: concavity changes

$\frac{d^2y}{dx^2}$ goes from $(+, 0, -)$, $(-, 0, +)$, $(+, und, -)$, OR $(-, und, +)$

First Derivative:

$f'(x) > 0$ function is increasing.

$f'(x) < 0$ function is decreasing.

$f'(x) = 0$ or DNE: Critical Values at x.

Relative Maximum: $f'(x) = 0$ or DNE and sign of $f'(x)$ changes from $+$ to $-$.

Relative Minimum: $f'(x) = 0$ or DNE and sign of $f'(x)$ changes from $-$ to $+$.

Absolute Max or Min:

MUST CHECK ENDPOINTS ALSO

The maximum value is a y-value.

Second Derivative:

$f''(x) > 0$ function is concave up.

$f''(x) < 0$ function is concave down.

$f'(x) = 0$ and sign of $f''(x)$ changes, then there is a point of inflection at x.

Relative Maximum: $f''(x) < 0$

Relative Minimum: $f''(x) > 0$

Write the equation of a tangent line at a point:

You need a slope (derivative) and a point.

$$y_2 - y_1 = m(x_2 - x_1)$$

Horizontal Asymptotes:

1. If the largest exponent in the numerator is < largest exponent in the denominator then $\lim_{x \to \pm\infty} f(x) = 0$.

2. If the largest exponent in the numerator is > the largest exponent in the denominator then $\lim_{x \to \pm\infty} f(x) = DNE$

3. If the largest exponent in the numerator is = to the largest exponent in the denominator then the quotient of the leading coefficients is the asymptote. $\lim_{x \to \pm\infty} f(x) = \frac{a}{b}$

ONLY FOUR THINGS YOU CAN DO ON A CALCULATOR THAT NEEDS NO WORK SHOWN:

1. Graphing a function within an arbitrary view window.
2. Finding the zeros of a function.
3. Computing the derivative of a function numerically.
4. Computing the definite integral of a function numerically.

Distance, Velocity, and Acceleration

$x(t)$ = position function
$v(t)$ = velocity function
$a(t)$ =acceleration function

The derivative of position (ft) is velocity (ft/sec); the derivative of velocity (ft/sec) is acceleration (ft/sec^2).

The integral of acceleration (ft/sec^2) is velocity (ft/sec) ; the integral of velocity (ft/sec) is position (ft).

Speed is | velocity |

If acceleration and velocity have the ***same sign***, then the speed is *increasing*..

If the acceleration and velocity have ***different signs***, then the speed is *decreasing.*

The particle is moving right when velocity is positive and particle is moving left when velocity is negative.

Displacement = $\int_{t_0}^{t_f} v(t)\, dt$

Total Distance = $\int_{initial\ time}^{final\ time} |\, v(t)|\ dt$

Average Velocity

$$= \frac{\text{final position} - \text{initial position}}{\text{total time}} = \frac{\Delta x}{\Delta t}$$

Accumulation = $x(0) + \int_{t_0}^{t_f} v(t)\, dt$

LOGARITHMS
Definition:

$$ln\ N = p \leftrightarrow e^p = N$$

$ln\ e = 1$
$\ln 1 = 0$
$\ln(MN) = \ln M + \ln N$
$\ln\left(\frac{M}{N}\right) = \ln M - \ln N$
$p \cdot \ln M = \ln M^p$

EXPONENTIAL GROWTH and DECAY:

When you see these words use: $y = Ce^{kt}$

"y is a differentiable function of t such that $y > 0$ and $y' = ky$ "

"the rate of change of y is proportional to y"

When solving a differential equation:

1. Separate variables first
2. Integrate
3. Add +C to one side
4. Use initial conditions to find "C"
5. Write the equation if the form of $y = f(x)$

"PLUS A CONSTANT"

The Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(b) - F(a)$$

Where $F'(x) = f(x)$

Corollary to FTC

$$\frac{d}{dx}\int_a^{g(u)} f(t)dt = f\big(g(u)\big)\frac{du}{dx}$$

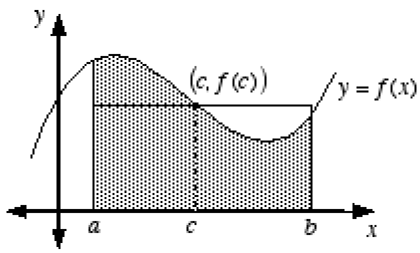
Mean Value Theorem for Integrals: The Average Value

If the function $f(x)$ is continuous on $[a, b]$ and the first derivative exists on the interval (a, b), then there exists a number $x = c$ on (a, b) such that

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{\int_a^b f(x)dx}{b-a}$$

This value $f(c)$ is the "average value" of the function on the interval $[a, b]$.

The rectangle has the same area as the shaded region under the curve.



Riemann Sums

A Riemann Sum means a rectangular approximation. Approximation means that you ***DO NOT EVALUATE THE INTEGRAL***; you add up the areas of the rectangles.

Trapezoidal Rule

For uneven intervals, may need to calculate area of one trapezoid at a time and total.

$A_{Trap} = \frac{1}{2}h[b_1 + b_2]$

For even intervals:

$$\int_a^b f(x)\,dx = \frac{b-a}{2n}\left[\begin{matrix} y_0 + 2y_1 + 2y_2 + \ldots \\ +2y_{n-1} + y_n \end{matrix}\right]$$

Values of Trigonometric Functions for Common Angles

θ	$sin\ \theta$	$cos\ \theta$	$tan\ \theta$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	"∞"
π	0	-1	0

Must know both inverse trig and trig values:

EX. $tan\ \frac{\pi}{4} = 1$ and $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

ODD and EVEN:

$\sin(-x) = -\sin x$ (odd)

$\cos(-x) = \cos x$ (even)

Trigonometric Identities

Pythagorean Identities:

$sin^2\theta + cos^2\theta = 1$

The other two are easy to derive by dividing by $\sin^2\theta$ or $\cos^2\theta$.

$1 + \tan^2\theta = sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

Double Angle Formulas:

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

Power-Reducing Formulas:

$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

Quotient Identities:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$

Reciprocal Identities:

$csc\ x = \frac{1}{sin\ x}$ or $\sin x\csc x = 1$

$sec\ x = \frac{1}{\cos x}$ or $\cos x\sec x = 1$

Basic Integrals

$$\int du = u + C$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + C \; n \neq -1$$

$$\int \frac{du}{u} = \ln|\, u \,| + C$$

$$\int e^u \, du = e^u + C$$

$$\int a^u \, du = \frac{a^u}{\ln a} + C$$

$$\int \sin u \, du = -\cos u + C$$

$$\int \cos u \, du = \sin u + C$$

$$\int \tan u \, du = -\ln|\cos u + C|$$

$$\int \cot u \, du = \ln|\sin u| + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C$$

$$\int \csc u \, du = -\ln|\csc u + \cot u| + C$$

$$\int \sec^2 u \, due = \tan u + C$$

$$\int \csc^2 u \, du = -\cot u + C$$

$$\int \sec u \tan u \, du = \sec u + C$$

$$\int \csc u \cot u \, du = -\csc u + C$$

Area and Solids of Revolution:

NOTE: (a, b) are x-coordinates and
(c, d) are y-coordinates

Area Between Two Curves:
Slices ⊥ to x-axis: $A = \int_a^b [f(x) - g(x)]\, dx$

Slices ⊥ to y-axis: $A = \int_c^d [f(y) - g(y)]\, dy$

Volume By Disk Method:

About x-axis: $V = \pi \int_a^b [R(x)]^2 \, dx$

About y-axis: $V = \pi \int_c^d [R(y)]^2 \, dy$

Volume By Washer Method:

About x-axis: $V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2)\, dx$

About y-axis: $V = \pi \int_c^d ([R(y)]^2 - [r(y)]^2)\, dy$

Volume By Shell Method:

About x-axis: $V = 2\,\pi \int_c^d y\, [R(y)]\, dy$

About y-axis: $V = 2\,\pi \int_a^b x\, [R(x)]\, dx$

General Equations for Known Cross Section where *base* is the distance between the two curves and *a* and *b* are the limits of integration.

SQUARES: $V = \int_a^b (base)^2 \, dx$

TRIANGLES

EQUILATERAL: $V = \frac{\sqrt{3}}{4} \int_a^b (base)^2 \, dx$

ISOSCELES RIGHT: $V = \frac{1}{4} \int_a^b (base)^2 \, dx$

RECTANGLES: $V = \int_a^b (base) \cdot h \, dx$
where h is the height of the rectangles.

SEMI-CIRCLES: $V = \frac{\pi}{2} \int_a^b (radius)^2 \, dx$
where radius is ½ distance between the two curves.

MORE DERIVATIVES:

$$\frac{d}{dx}\left[\sin^{-1}\frac{u}{a}\right] = \frac{1}{\sqrt{a^2-u^2}}\,\frac{du}{dx}$$

$$\frac{d}{dx}\left[\cos^{-1}x\right] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\tan^{-1}\frac{u}{a}\right] = \frac{a}{a^2+u^2}\,\frac{du}{dx}$$

$$\frac{d}{dx}\left[\cot^{-1}x\right] = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}\left[\sec^{-1}\frac{u}{a}\right] = \frac{a}{|u|\sqrt{u^2-a^2}}\,\frac{du}{dx}$$

$$\frac{d}{dx}\left[\csc^{-1}x\right] = \frac{-1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left(a^u\right) = a^u \ln a\,\frac{du}{dx}$$

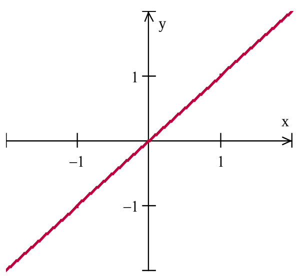
$$\frac{d}{dx}\left[\log_a x\right] = \frac{1}{x\ln a}$$

MORE INTEGRALS:

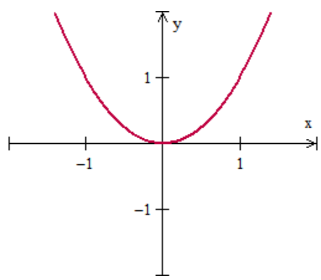
$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

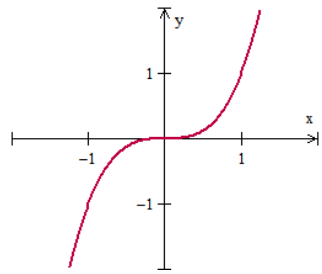
$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{|u|}{a} + C$$



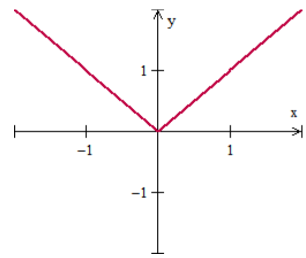
$y = x$



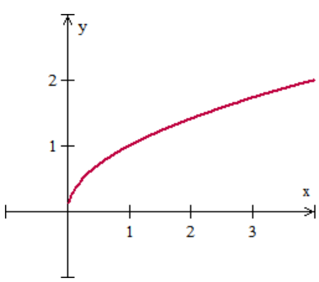
$y = x^2$



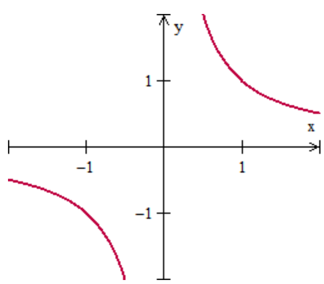
$y = x^3$



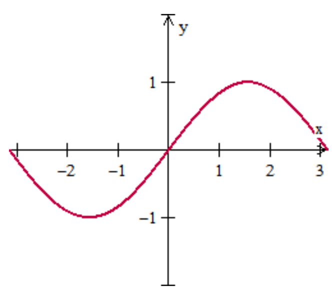
$y = |x|$



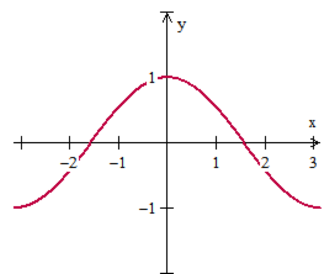
$y = \sqrt{x}$



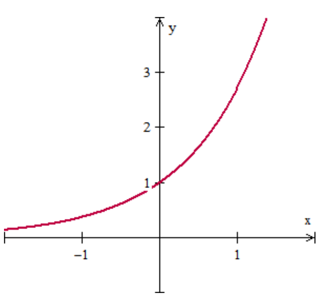
$y = \frac{1}{x}$



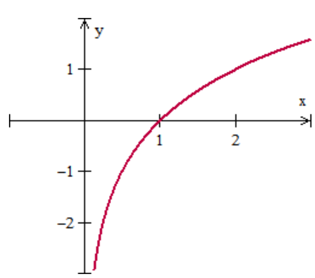
$y = \sin x$



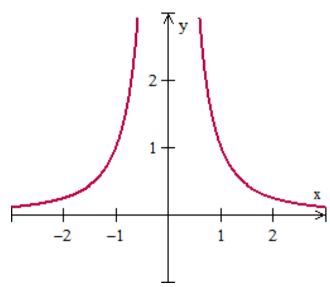
$y = \cos x$



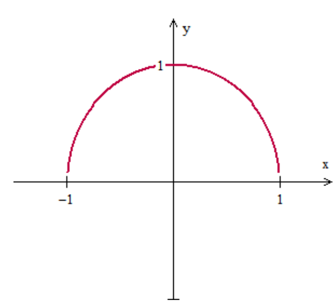
$y = e^x$



$y = \ln x$



$y = \frac{1}{x^2}$



$y = \sqrt{a^2 - x^2}$